MATHS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 18	19 – 35	36 – 52	53 – 63	64 – 75	76 – 87	88 – 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 13	14 – 19	20 – 23	24 – 28	29 – 33	34 – 40

The range and suitability of the work submitted

The vast majority of schools selected tasks from the set of tasks offered by the IB. In a few cases HL tasks were presented, or teachers had modified the IB tasks in some way. It is critical that teachers submit solution keys and assessment expectations for tasks so that the rationale for the assessment is evident.

Many schools did not submit background information as to previous knowledge of the topics used in the task, or the availability and expectations of use of technology. In order for the moderators to be able to confirm teacher marks it is important that they understand the expectations of the classroom teachers.

Candidate performance against each criterion

Criterion A

There are still many cases where inappropriate notation is not being penalized. Use of calculator/computer notation will generally preclude a level 2. Candidates and teachers are lax in addressing issues of accuracy with proper notation. If answers are approximate then some form of approximately equals symbol is necessary. In modelling tasks candidates often use the same variable (usually '*y*') for multiple model functions. This leads to confusion when comparisons are made and should be avoided. Candidates should be taught to use subscripted variables to distinguish between distinct models for the same behaviour. Candidates sometimes used incorrect terminology (e.g. "exponential" for "quadratic").

Criterion B

The use of graphing software has allowed candidates to create clear and useful graphs that are well-labelled. Few candidates offered work in a question-and-answer format. While the overall quality was quite good there were many gaps in the explanations offered. In some cases an “explanation” was offered after a result was given, suggesting that the candidate had worked backwards from a result that was already known. This was particularly true for Type I tasks. Candidates should learn that long drawn-out explanations are not helpful and that a feature of good communication is the ability to present ideas in a concise manner.

Criteria C and D

In the Type I tasks candidates were quite successful in attaining level 4 for C. There were, however, cases where a cogent analysis was missing yet the desired result appeared. In other cases general statements were proposed after investigating just one case. Most candidates continue to have difficulty properly validating their conjectures. Often they simply substituted values of the variable (say *n*) into their proposed general statement, and showed that the result matched the data they used to generate the statement in the first place. They must learn that they should be checking the results against the original pattern of behaviour and use additional values rather than those already found. Teachers are reminded that a **formal** explanation (e.g. an algebraic proof) is sufficient for level 5 for C and D, and that no further testing is required. The consideration of scope and limitations is often brief and superficial. Candidates should be encouraged to imagine all possibilities, including negatives, rationals, and irrationals, and to test these using the power of their graphic display calculators (GDCs). Very few candidates were able to provide adequate explanations that would result in the award of D5.

In the Type II tasks most candidates adequately identified variables and considered some constraints. Identification of parameters often appeared as a part of the analysis, but their role was not made clear. Fewer candidates chose to develop a model function by using a series of graphical transformations. When discussing the quality of fit many candidates offered superficial comments. While a quantitative analysis of fit is not required at SL, there should be some significant comments on how well the model matches the data. Applying the model function to a new set of data and commenting on the quality of fit is sufficient for C5. The necessary modifications to improve the fit are rewarded in D5.

Some candidates used regression analysis on a GDC or computer as the primary tool for model development. Some of these first found a model function by regression techniques, then went back and “analysed” the situation to develop much the same model function. Teachers and candidates are reminded that this approach can achieve only a maximum of C2 since the requisite analytical steps are based on information already obtained by calculator or computer.

In the Population Trends in China task, the data certainly suggested a linear model. However, the study of population models suggests that this is simply not reasonable over a long period of time. Thus other models should have been considered as well in order to attain higher marks in criterion C.

The most important aspects of criterion D for Type II tasks are interpretation in context, and consideration of reasonableness and accuracy. These were not well addressed by the majority of candidates. Some focussed on a purely mathematical interpretation and scored no higher than D2. Others offered only a superficial connection between their proposed model function and the reality of population or g-force tolerance on the human body. Most did not explicitly consider the importance of accuracy, often not using use any form of approximately equals notation, nor discussing the effect of using less or more accurate parameters in their functions. The award of D5 should be restricted to work that has critically considered these issues.

Criterion E

Candidates made good use of graphing technology in terms of creating graphs. However, the true value of the graphs within the work varied immensely. While it is more difficult to find ways to use technology in Type I activities it is certainly possible to better exploit the graphing of the data generated to explore, confirm, and predict. Some candidates recognized that matrix methods could be used to generate the quadratic relations in the Stellar Numbers task, using their GDC in the process. Some used spreadsheets effectively for the Infinite Summation task, although it was not always clear whether they had provided actual output or if they had transferred spreadsheet data to a table. Many were able to find software on the internet that would draw clear diagrams for the *p*-stellar shapes. While this made their work neater it did not in itself produce a higher mark under criterion E.

Criterion F

Most candidates achieved level 1. This recognizes that the candidate made a serious attempt to complete the task to the best of their ability. F0 should be awarded only in cases where it is clear that little attempt was made to complete the task, and the work is essentially unacceptable. F2 is reserved for work that has addressed all aspects of the task, and has demonstrated insight, precision, and significant understanding. The work should be truly admirable and not simply judged against the normal quality of the candidate’s work.

Recommendations for the teaching of future candidates

It is essential that the teacher work through the task before assigning it to the class. In this way expectations can be identified and guidance offered to candidates can assist in helping them find their way through to a successful presentation. There exist now many IB tasks that are not available for submission but which can be used for practice. These also provide practice for teachers in the development of standardization matrices or solution keys that clearly describe the expectations for each assessment level.

Working through tasks and creating solution keys and standardization matrices will also identify issues related to issues such as appropriate notation, quality and nature of explanation, types of analyses, use of technology, and considerations worthy of F2. The expectations developed for the practice tasks can be shared with candidates to better explain what is expected under each criterion level.

Teachers should discuss in classes how a conjecture can be validated and how a model function can be interpreted in context. Examples of resourceful use of technology in the context of the task can be shared. Candidates should be shown, for example, how graphical transformations might be used to fit a model function, and how an appropriate series of graphs demonstrating the evolution of the eventual model can be shown.

Teachers should ensure that they read recent subject reports to get an idea of what issues exist regarding strengths and weaknesses in assessment.

Further comments

Teachers should ensure that all requisite forms are properly completed and submitted with the sample. Background information and solution keys greatly assist in the moderation process.

External assessment

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 40	41 – 52	53 – 65	66 – 77	78 – 90

The areas of the programme and examination that appeared difficult for the candidates

- Probabilities involving more than one event
- Discrete probability distributions
- Working with double-angle formulas
- Using the discriminant to determine the nature of the roots of a quadratic
- Finding the parameters of a trigonometric function
- Chain rule differentiation
- Finding the equation of the tangent to a curve at a point
- Rules of logarithms

The levels of knowledge, understanding and skill demonstrated

Most candidates were able to make some sort of attempt on each question. There were fewer questions left unanswered than in previous examinations. Some topics were very well done by the majority of candidates:

- vertex form of a quadratic function
- working with 2×2 matrices
- transformations of functions
- simple probability
- integration of basic polynomials
- using sine, cosine, and the Pythagorean identity

- finding simple vectors, and the vector equation of a line in 3 dimensions
- algebraic manipulation

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A surprising number of candidates missed part (a) of this question, which required them to write the equation of the axis of symmetry. Some candidates did not write their answer as an equation, while others simply wrote the formula $x=-\frac{b}{2a}$. The rest of this question was answered correctly by the large majority of candidates. The mistakes seen in part (c) were generally due to either incorrect substitution of a point into the equation, or substitution of the vertex coordinates, which got the candidates nowhere.

Question 2

This question involving 2×2 matrices was answered correctly by the large majority of candidates. In part (a), there were a few candidates who did not know the correct method for multiplying matrices. In part (b), there were a few candidates who mistakenly found the inverse of their matrix from part (a), rather than finding the inverse of matrix ***P***.

Question 3

Candidates did very well on parts (a) and (b) of this probability question, and knew to multiply the probabilities of independent events in part (b). However, in part (c), very few candidates considered that there are two ways to draw one red and one blue marble, and therefore did not earn full marks on this question. There were also some candidates who tried to add, rather than multiply, the probabilities in parts (b) and (c).

Question 4

This question, which required candidates to integrate a simple polynomial and then substitute an initial condition to solve for "*c*", was very well done. Nearly all candidates who attempted this question were able to earn full marks. The very few mistakes that were seen involved arithmetic errors when solving for "*c*", or failing to write the final answer as the equation of the function.

Question 5

Candidates generally earned either full marks or only one mark on this question. The most common error was where candidates only wrote the equation for $\mathrm{E}(X)=1.7$, and tried to rearrange that equation to solve for *q*. The candidates who also knew that the sum of the probabilities must be equal to 1 were very successful in solving the resulting system of equations.

Question 6

While the majority of candidates knew to use the Pythagorean identity in part (a), very few remembered that the cosine of an angle in the second quadrant will have a negative value. In part (b), many candidates incorrectly tried to calculate $\tan 2\theta$ as $2\times\tan\theta$, rather than using the double-angle identities.

Question 7

A good number of candidates were successful in using the discriminant to find the correct values of *k* in part (a), however, there were many who tried to use the quadratic formula without recognizing the significance of the discriminant. Part (b) was very poorly done by nearly all candidates. Common errors included finding the wrong values for *k*, and not realizing that there were 11 possible values for *k*.

Question 8

In part (a), nearly all the candidates correctly found the vector PQ, and the majority went onto find the correct vector equation of the line. There are still many candidates who do not write this equation in the correct form, using "$\boldsymbol{r} =$", and these candidates were penalized one mark. In part (b), the majority of candidates knew to set the scalar product equal to zero for the perpendicular vectors, and were able to find the correct value of *p*. A good number of candidates used the correct method to find the intersection of the two lines, though some algebraic and arithmetic errors kept some from finding the correct final answer.

Question 9

Part (a) of this question proved challenging for most candidates. Although a good number of candidates recognized that the period was 8 in part (b), there were some who did not seem to realize that this period could be found using the given coordinates of the maximum and minimum points. In part (c), not many candidates found the correct derivative using the chain rule. For part (d), a good number of candidates correctly set their expression equal to -2π, but errors in their previous values kept most from correctly solving the equation. Most candidates who had the correct equation were able to gain full marks here.

Question 10

While most candidates answered part (a) correctly, finding the equation of the tangent, there were some who did not consider the value of their derivative when $x = 4$. In part (b), most candidates knew that they needed to integrate to find the area, but errors in integration, and misapplication of the rules of logarithms kept many from finding the correct area.

In part (c), it was clear that a significant number of candidates understood the idea of the reflected function, and some recognized that the integral was the negative of the integral from part (b), but only a few recognized the relationship between the areas. Many thought the area between *h* and the *x*-axis was 120.

Recommendations for the teaching of future candidates

As always, teachers need to be sure their students are exposed to all areas of the syllabus. It was apparent that this is not always the case, though some areas are covered consistently well, as was evidenced by the large number of candidates earning full marks on question 4.

It is a good idea for candidates to be familiar with examination-style questions, and to practice working under timed conditions. It seems that some candidates were hurrying to finish the questions at the end, and there were a few candidates who left parts of the final questions unanswered.

It is always important for candidates to present their work in a neat, organized manner. This is especially true with e-marking. When the scripts are scanned, all writing will show up as dark black, even if this writing is "scratch work" or unintentional stray marks. This often makes it difficult to decipher the candidates' intended working and answers.

Finally, it is pleasing to note that candidates in this session did a nice job of showing their work and demonstrating the methods they used. As a result, many were able to earn at least partial marks on each question they attempted. Candidates should be encouraged to clearly show their working in future examinations.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 19	20 – 39	40 – 55	56 – 64	65 – 72	73 – 81	82 – 90

The areas of the programme and examination that appeared difficult for the candidates

- Combining transformations of functions (especially stretch parallel to the *y*-axis)
- Binomial probability
- Ambiguous case of sine rule
- Solving equations that involve logarithms
- Recognition of integration of velocity to find distance
- 'Show that' questions
- Interpreting the second derivative as a rate of change

The levels of knowledge, understanding and skill demonstrated

Candidates demonstrated a good level of knowledge and understanding with most topics. Strengths included:

- sequences and series
- arc length and area of sector
- sketching the graph of a function using the graphing calculator
- the trigonometry of right-angled triangles
- normal distribution.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 - Composite and inverse functions

All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

Question 2 - Statistics

Parts (a) and (b) were generally well done. Some candidates could only earn the first mark in part (c) for finding 82% of 200. Others gave the answer as 164, neglecting to subtract this value from the total of 200.

Question 3 - Length of an arc and area of a sector

Part (a) was almost universally done correctly. Many also had little trouble in part (b), with most subtracting from the circle's area, and a minority using the reflex angle. A few candidates worked in degrees, although some of these did so incorrectly by using the radian area formula. Some candidates only found the area of the unshaded sector.

Question 4 - Trigonometry in non right-angled triangles

Most candidates were comfortable applying the sine rule, although many were then unable to find the obtuse angle, demonstrating a lack of understanding of the ambiguous case. This precluded them from earning marks in part (b). Those who found the obtuse angle generally had no difficulty with part (b).

Question 5 - Finding a specific term in a binomial expansion

Many candidates were familiar with the binomial expansion, although some expanded entirely which at times led to careless errors. Others attempted to use Pascal's Triangle. Common errors included misidentifying the binomial coefficient corresponding to this term and not squaring the 3 in $(3x)^2$.

Question 6 - Exponential function

For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analyzing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equaled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

Question 7 - Transformations of functions and kinematics

While a number of candidates had an understanding of each transformation, most had difficulty applying them in the correct order, and few obtained the completely correct answer in part (a). Many earned method marks for discerning three distinct transformations. Few candidates knew to integrate to find the distance traveled. Many instead substituted time values into the velocity function or its derivative and subtracted. A number of those who did recognize the need for integration attempted an analytic approach rather than using the GDC, which often proved unsuccessful.

Question 8 - Sequences

Most candidates found part (a) straightforward, although a common error in (a)(ii) was to calculate 40 divided by $\frac{1}{2}$ as 20. In part (b), some candidates had difficulty with the “show that” and worked backwards from the answer given. Most candidates obtained the correct equation in part (c), although some did not reject the negative value of n as impossible in this context.

Question 9 - Binomial and normal probability

Many stronger candidates were completely successful with this question, employing technology efficiently. A number of candidates did not recognize the binomial probability in parts (a) and (b), and in part (b) a proportion of candidates just subtracted their part (a) answer from one. Candidates had more success with the normal distribution and many obtained follow-through marks in part (e) after an error made in part (b). Many candidates did not appreciate the independence in part (e) and added probabilities rather than multiplying them. A number of candidates were penalized for not giving their answers to 3 significant figures.

Question 10 - Differential calculus

Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value. Most had a valid approach in part (c) using either the quotient or product rule, but many had difficulty applying the chain rule with a function involving e and simplifying. Part (d) was difficult for most candidates. Although many associated rate of change with derivative, only the best-prepared students had valid reasoning and could find the correct interval with both endpoints.

Recommendations and guidance for the teaching of future candidates

- While many candidates demonstrated a good use of the GDC, a number are still using analytical methods where a GDC approach would be more efficient. Teachers should ensure that their students feel confident using their calculators in such situations.
- Some candidates did not label the sub-parts of questions. When the examiner does not know which part of a question the candidate is trying to answer, the candidate may not earn all the marks that he should. Teachers should encourage students to label each part of their answer with the correct sub-heading.
- Teachers should continue to emphasize the need for candidates to present work clearly.
- Candidates need to understand that working backwards from the given result in a “show that” question is not acceptable.

- Candidates should only use graph paper for drawing graphs and not for doing other sub-parts of a problem, as work on scanned graph paper is difficult to read and mark.
- Candidates should be aware that graphing a first derivative in the GDC is an efficient method for finding the x-coordinate of the point of inflexion.
- Candidates need to be aware that when applying multiple transformations to a function, there is an appropriate order.
- To maintain accuracy, candidates need to avoid premature rounding.